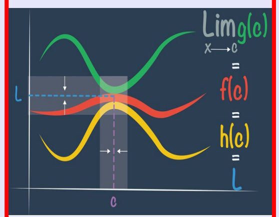
Calculus I

Lecture 3



Feb 19-8:47 AM

Square - Root Function

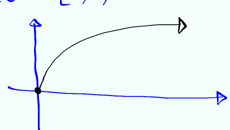
$f(x) = \sqrt{x}$

Y-Int $\quad f(0) = 0 \quad (0,0)$

x-Int $\quad f(x) = 0 \quad \sqrt{x} = 0 \quad x = 0 \quad (0,0)$

Domain $\quad$ Radicand $\geq 0 \quad x \geq 0 \quad [0, \infty)$

Range $\quad \sqrt{x} \geq 0 \quad [0, \infty)$



Difference quotient $\frac{f(x+h) - f(x)}{h}$, evaluate for $h=0$.

For $h=0$

$$\frac{\sqrt{x+0} - \sqrt{x}}{0} = \frac{\sqrt{x} - \sqrt{x}}{0} = \frac{0}{0} \quad \text{I.F.}$$

$$\frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

Let's evaluate for $h=0$

$$\frac{1}{\sqrt{x+0} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

$x > 0$

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Absolute Value Function

$f(x) = |x|$

Y-Int $(0,0)$

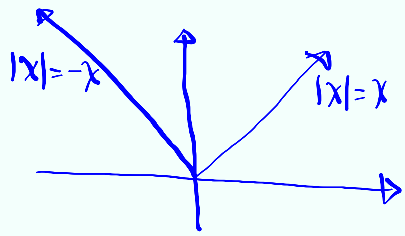
x-Int $(0,0)$

Domain $(-\infty, \infty)$

Range $[0, \infty)$

If $x \geq 0 \rightarrow |x| = x$

If $x < 0 \rightarrow |x| = -x$



$$f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

Piece-wise Function

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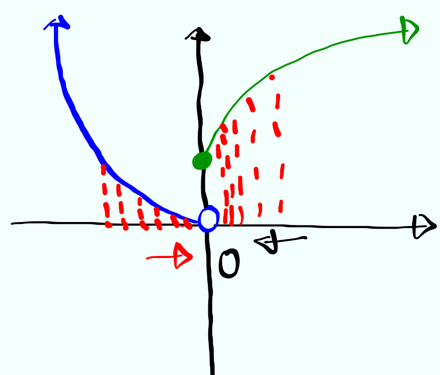
Consider the function below

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ \sqrt{x} + 1 & \text{if } x \geq 0 \end{cases}$$



Piece-wise Function

As $x \rightarrow 0^+$, $f(x) \rightarrow 1$ $\Rightarrow \lim_{x \rightarrow 0^+} f(x) = 1$

As $x \rightarrow 0^-$, $f(x) \rightarrow 0$ $\Rightarrow \lim_{x \rightarrow 0^-} f(x) = 0$

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$f(x) = \dfrac{1}{x-1}$

1) Y-Int $(0,-1)$

2) x-Int None

$y=0$

$\dfrac{1}{x-1} = 0$

No Solution

3) Domain

$x-1 \neq 0$

$x \neq 1$

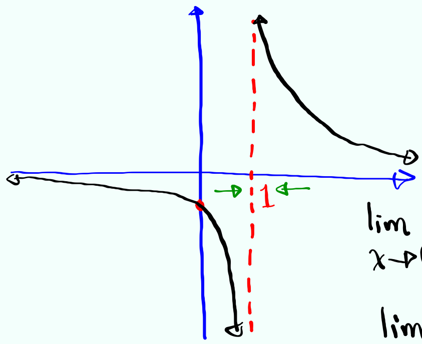
$(-\infty, 1) \cup (1, \infty)$

Vertical Asymptote

$x = 1$

Range

$(-\infty, 0) \cup (0, \infty)$



$\lim\limits_{x \to \infty} f(x) = 0$ $\quad$ $\lim\limits_{x \to -\infty} f(x) = 0$

$\lim\limits_{x \to 1^+} f(x) = \infty$ $\quad$ $\lim\limits_{x \to 1^-} f(x) = -\infty$

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Evaluate $\dfrac{f(x+h) - f(x)}{h}$ for $h=0$.

$$\frac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h}$$

For $h=0$

$$\frac{\frac{1}{x+0-1} - \frac{1}{x-1}}{0} = \frac{0}{0} \text{ I.F.}$$

LCD: $(x+h-1)(x-1)$

$$\frac{\cancel{(x+h-1)}(x-1) \cdot \frac{1}{\cancel{x+h-1}} - (x+h-1)\cancel{(x-1)} \cdot \frac{1}{\cancel{x-1}}}{(x+h-1)(x-1) \cdot h}$$

$$= \frac{(x-1) \cdot 1 - (x+h-1) \cdot 1}{h(x+h-1)(x-1)} = \frac{\cancel{x}\cancel{-1} \cancel{-x} - \cancel{h} \cancel{+1}}{\cancel{h}(x+h-1)(x-1)}$$

-1

$$= \frac{-1}{(x+h-1)(x-1)}$$

For $h=0$

$$\frac{-1}{(x+0-1)(x-1)} = \frac{-1}{(x-1)^2}$$

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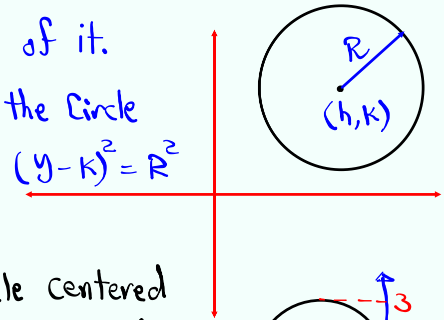
Circle

A shape that all its points is the same distance from a fixed point which is the center of it.



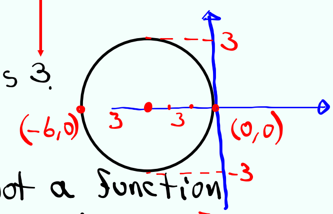
Equation of the Circle

$(x-h)^2 + (y-k)^2 = R^2$

Draw a circle centered at $(-3,0)$ with radius 3.

$(x-(-3))^2 + (y-0)^2 = 3^2$

$(x+3)^2 + y^2 = 9$



Not a function

Domain $[-6,0]$

Range $[-3,3]$

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Consider $f(x) = \sqrt{6x - x^2} \geq 0$

Range $[0,3]$

Domain $[0,6]$

Y-Int $(0,0)$

X-Int $(0,0)$ → $y=0$

$(6,0)$

$f(x)=0$

$\sqrt{6x-x^2}=0$

$6x-x^2=0^2$

$x(6-x)=0$ → $x=0$, $x=6$

1) Replace $f(x)$ with Y. $y = \sqrt{x-x^2}$

2) Square both sides $y^2 = 6x - x^2$

3) What do you recognize?

$y^2 - 6x + x^2 = 0$

$\underbrace{x^2-6x+9}+y^2 = 0 + 9$

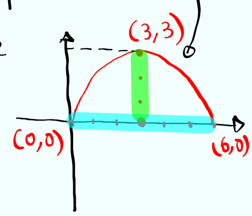
$(x-3)^2 + (y-0)^2 = 3^2$

Circle

Center at $(3,0)$

Radius 3

$f(x) = \sqrt{6x-x^2}$



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